Weirdos From Another Dimension!

Introduction to Fractals

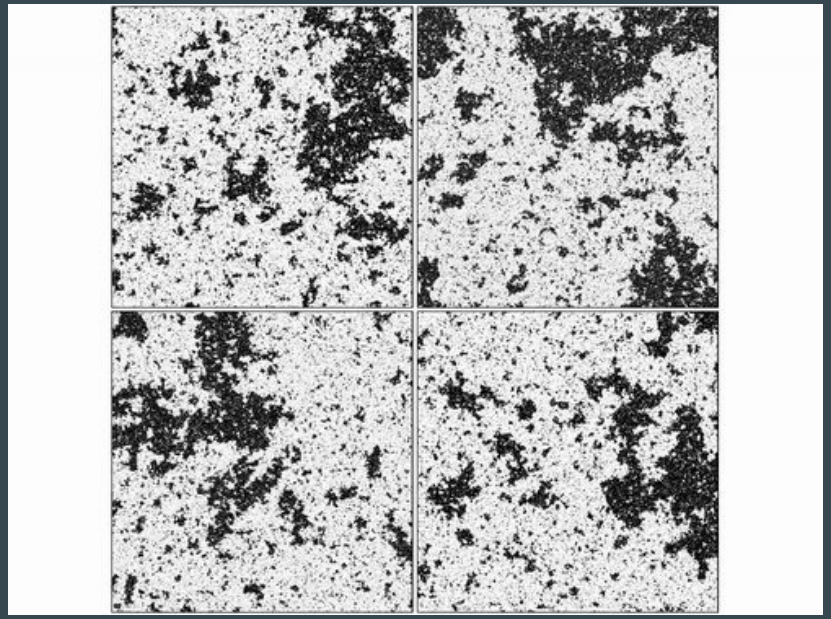
Fractals: Relevance to ABM

The complex emergent behavior of agent-based models is often scale-invariant and/or self-similar

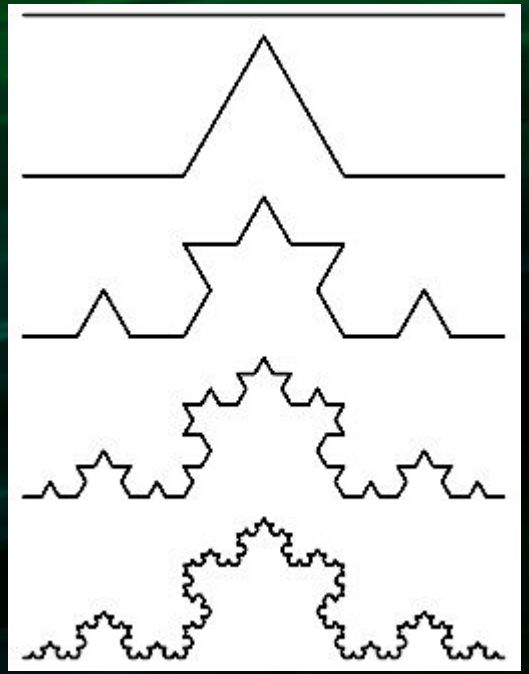
Recall: Ising at critical temperature

We should try to understand these incredibly convoluted outcomes!

The same patterns can come from simpler systems, which can give us insights into our ABM's emergent dynamics

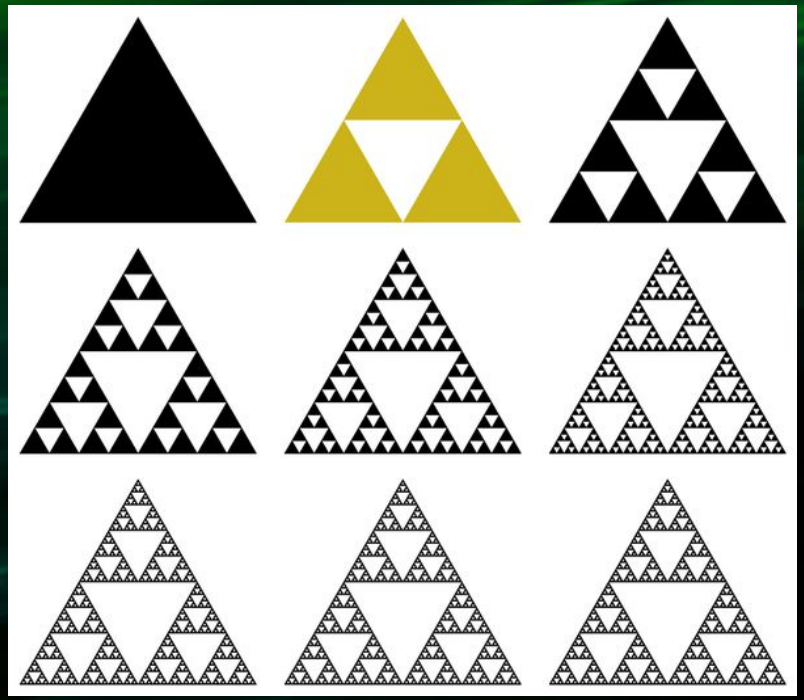


Koch Curve



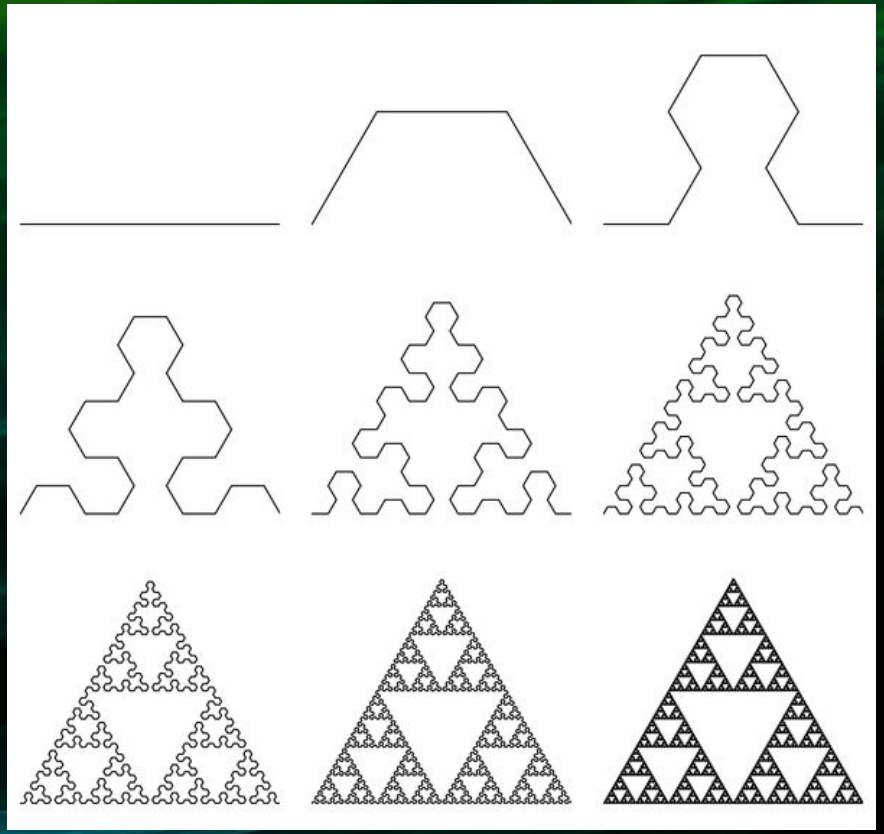
What's this thing's length?

Sierpinski Triangle



What's this thing's area?

Still Sierpinkski Triangle?



What's *this* thing's length?

What's going on?

What is Dimension?

Here, we'll say it's how the amount of 'stuff' scales when we scale all lengths by some amount

- Line: x2 stretch ->x2 stuff, x3 stretch->x3 stuff, … (just the scaling factor)
- Square: x2->x4 stuff, x3->x9 stuff, … (square the scaling factor)
- Cube: x2->x8 stuff, x3->x27 stuff, … (cube the scaling factor)
- Hypercube (Tesseract): ?

Any other shape in 1, 2, 3, etc dimensions shares the same rule, so they share the same dimension

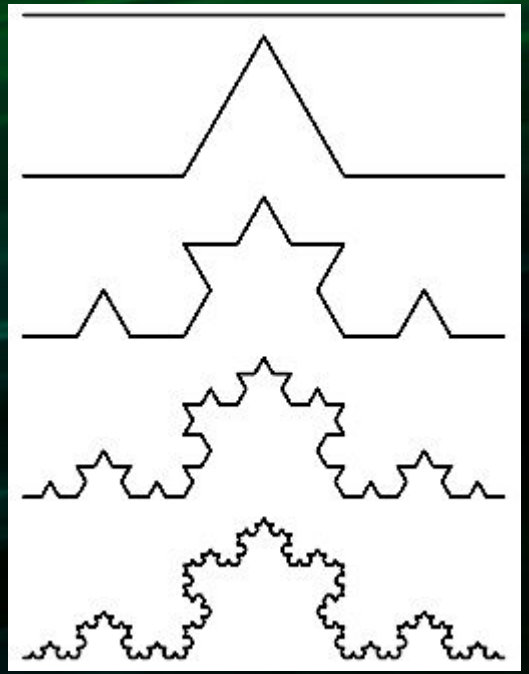
What is Dimension?

How to unify this notion?

D = log(stuff multiplier) / log(stretch factor)

Bonus fact: this “similarity dimension” (closely related to “box-counting dimension”) is an upper bound on the more robust Hausdorff Dimension

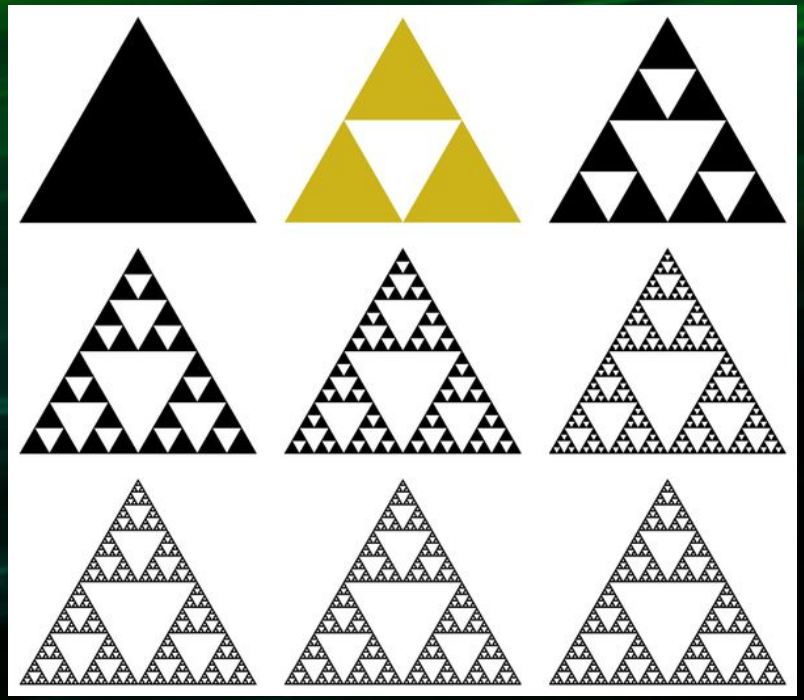
Koch Curve



What's this thing's dimension?

Log(4)/Log(3) = 1.262...

Sierpinski Triangle



What's this thing's dimension?

Log(3)/Log(2) = 1.585...

Iterated Function Systems

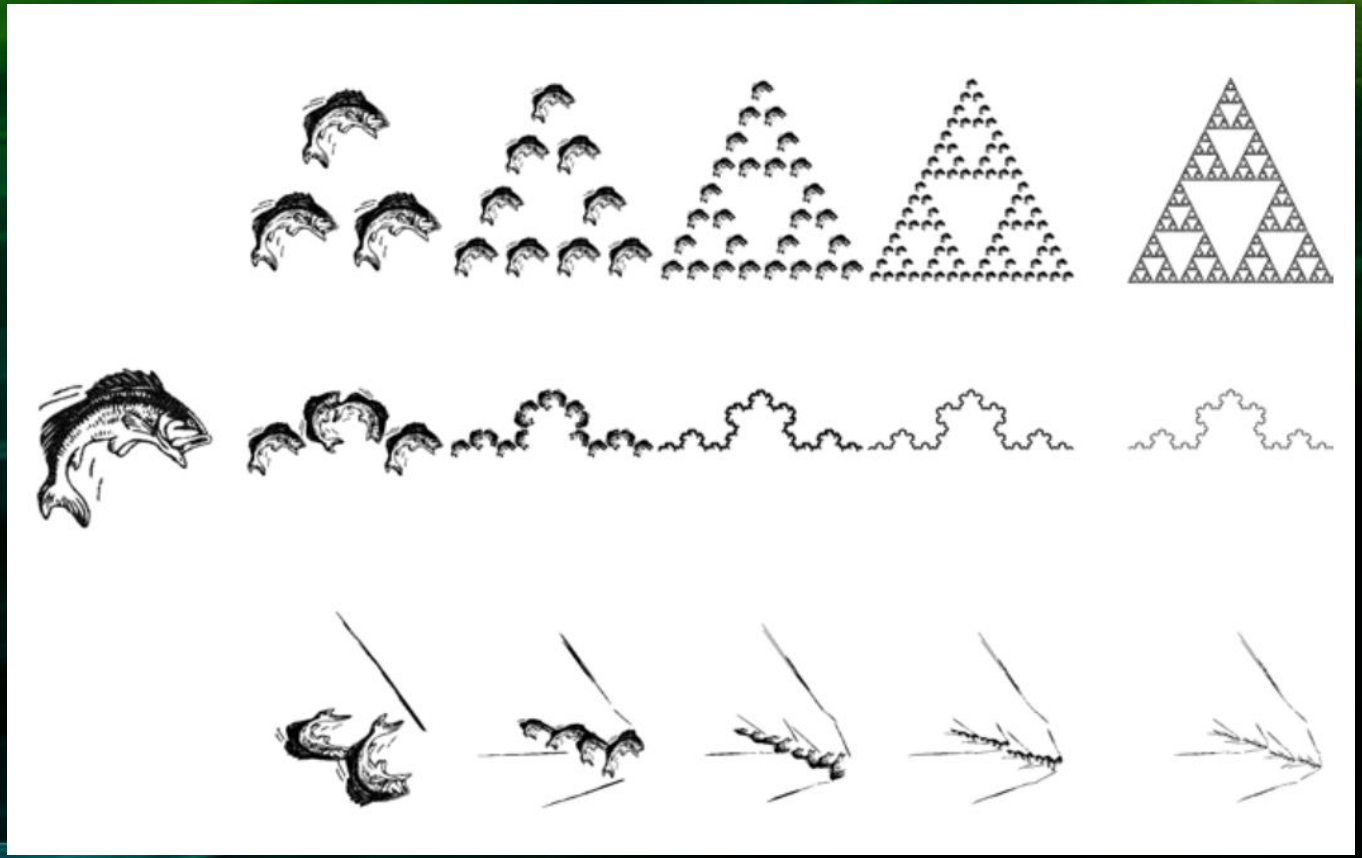
In general, we set up a set of rules:

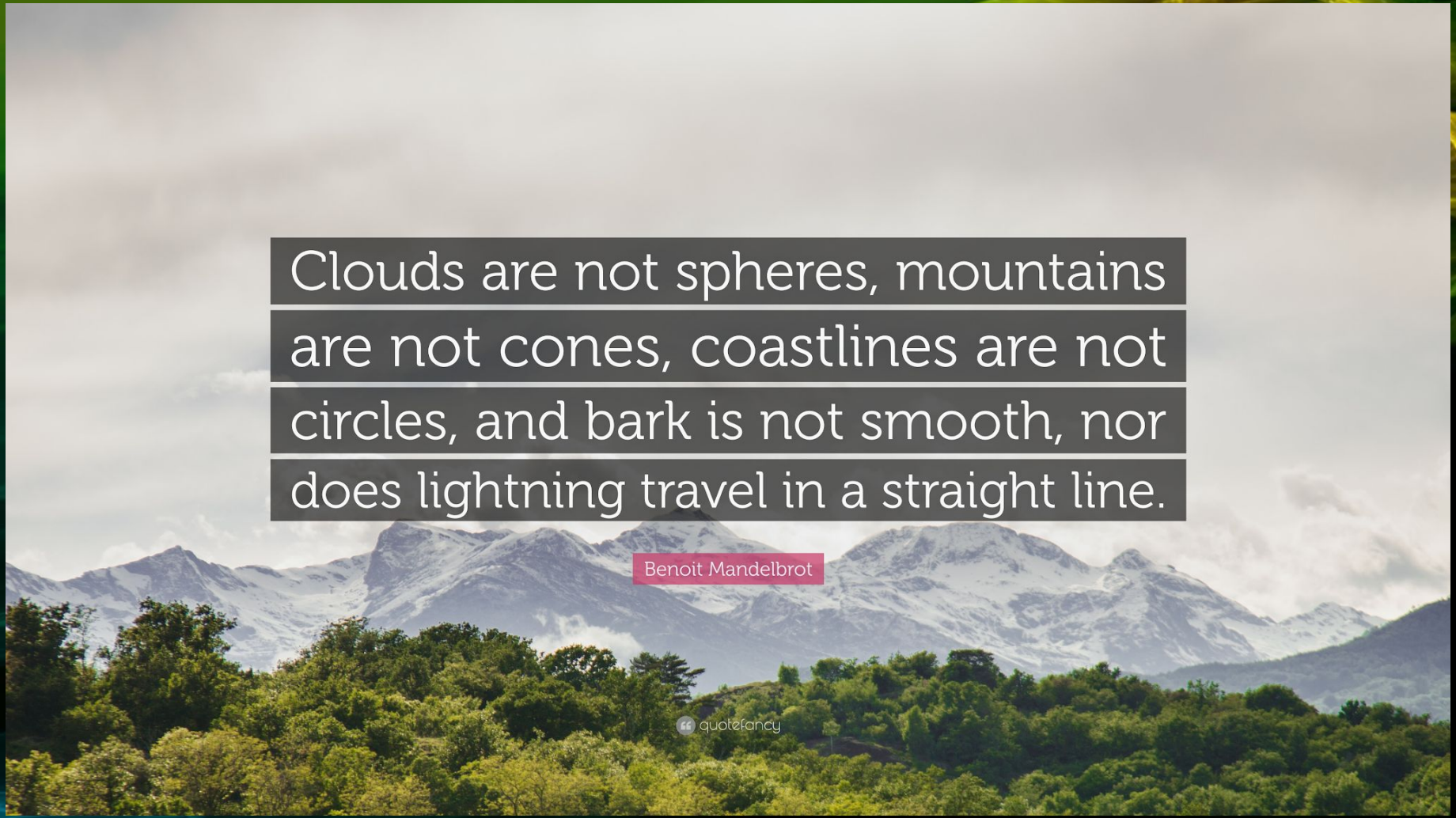
- How many copies?
- Scaled how much?
- Rotated how much?
- Placed where?

Scaling can even be different in each direction (“Affine”)...

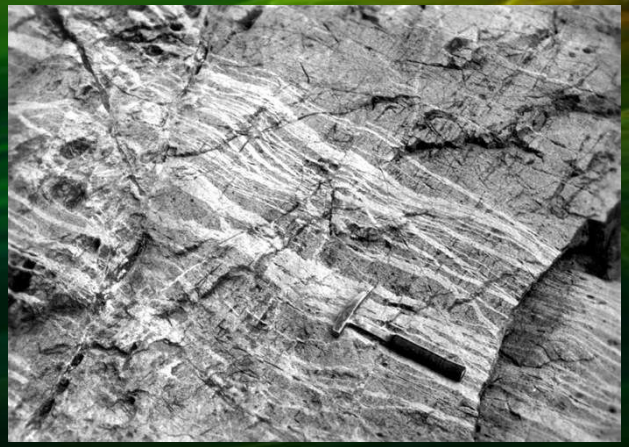
When you repeatedly apply this process, the result will start to look the same every step! This is called the “attractor”. This attractor is the true fractal.

It doesn’t matter what you start with!




Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line.
Benoit Mandelbrot
quotefancy

Craters



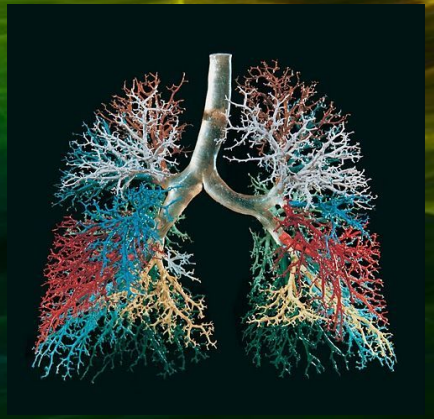
Rocks

Frost

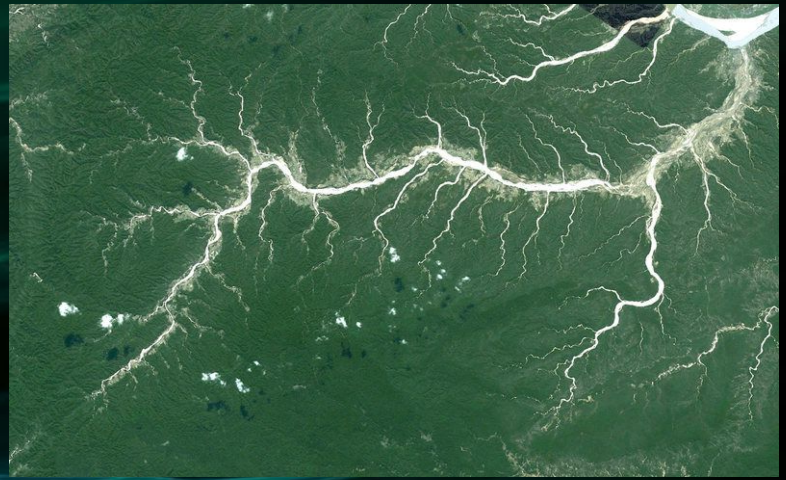
Clouds

Trees

Desert Drainage



Lungs (also blood vessels)



Rivers

Lightning

Aloe

Kale

Romanesco Broccoli

Natural “Fractals”

None of these things are “real” fractals--they’re ultimately 3-dimensional.

BUT, their fractal-like self-similarity:

- hints at processes that cause them
- gives them valuable properties
  - anything that a part does, the whole does.
    (E.g., water transport)

It’s kind of spooky to recognize these real-world things from a simple IFS: fern, frost, moss/forest

Natural “Fractals”: Why?

Short answer: the same process happens at different length scales

For clouds, a large gust of wind or patch of mist behaves a lot like a small one.

> “Big whorls have little whorls,
> Which feed on their velocity;
> And little whorls have lesser whorls,
> And so on to viscosity”

Satellite/radar images of clouds: (2-D) From 1 sq kilometer to 1.2 million sq km, perimeter dimension is 1.35!

LOVEJOY, S. (1982). Area-Perimeter Relation for Rain and Cloud Areas *Science, 216* (4542), 185-187 DOI: 10.1126/science.216.4542.185

Natural “Fractals”: Why?

Living self-similarity is observed all over the place, hinting at how and why they grow.

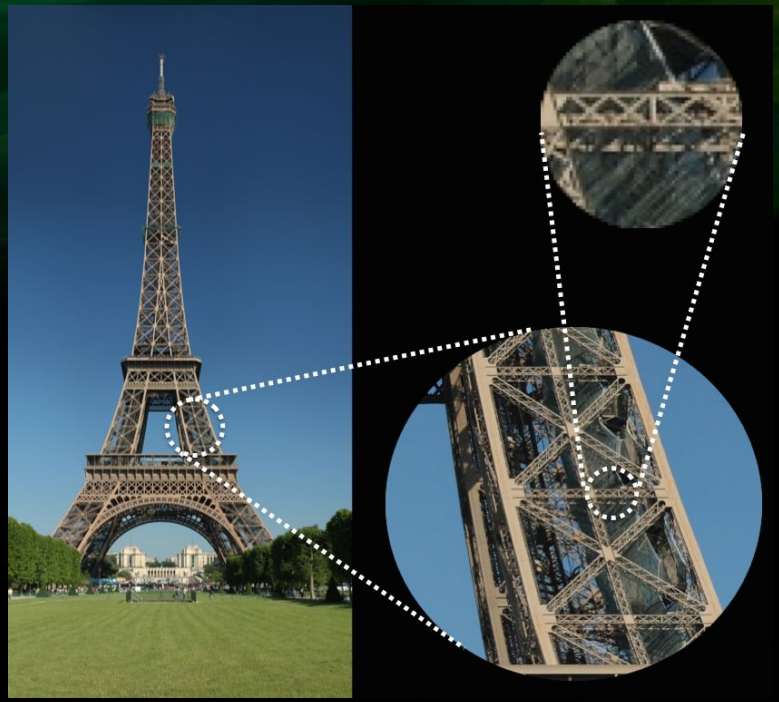
Self-similar constructions can create extremely complex and functional structures from very simple rules.

This makes it great for genetic coding of growth patterns: Simple rules let microscopic processes scale up easily.

For example, each part of a fern needs only a few signals in order to know how to grow, and they can all follow the same rules!

Applications: Structural Engineering

Replacing a beam with a truss improves the strength-to-weight ratio. Repeat!



Applications: Image compression

Can find an IFS whose attractor is the image in question

Takes a while to compress (zip), but unzips very quickly

.FIF = “fractal image format”

Much smaller than .BMP, usually smaller than .JPG, but this isn’t what real zip programs use. (see: Information Theory)

Loses/changes information, but generates more “detail” no matter the zoom level

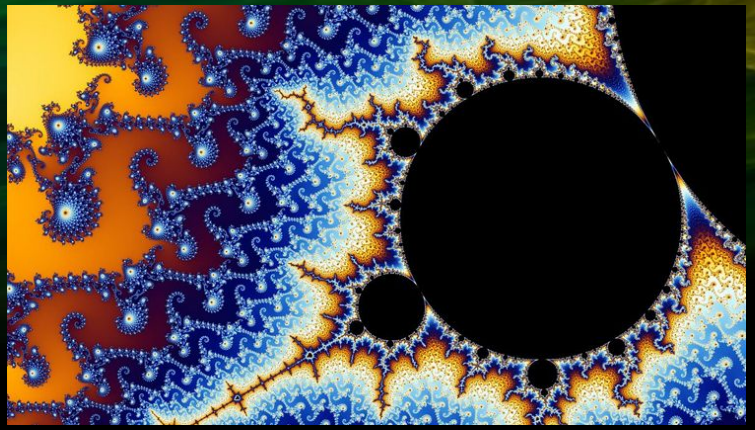
Compressed, then rebuilt to be more pixels VS real image, zoomed

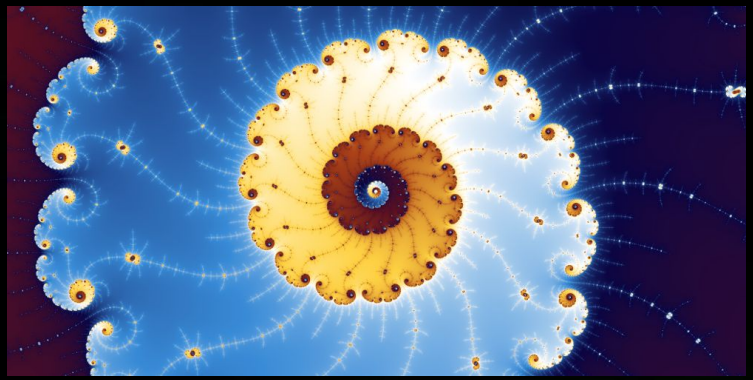
Beyond Self-similarity

Strictly self-similar fractals are an EXTREMELY SMALL subset of all fractals. Most are much weirder!

When mathematicians use infinite processes to define things, they often end up being fractals

In a certain sense, “most” are undefinable/unreproducible (e.g. ~clumpy random noise)





The Mandelbrot Set

Is awesome

The Mandelbrot Set: Background

Addition in the plane: Add x and y coordinates

Multiplication in the plane: Multiply x and y coordinates

The Mandelbrot Set: Background

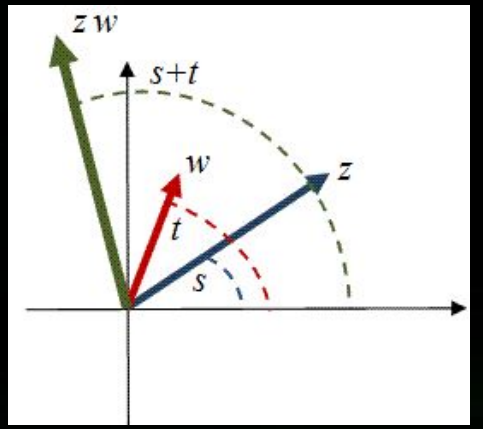
Addition in the plane: Add x and y coordinates

Multiplication in the plane: ~~Multiply x and y coordinates~~

NO! (nonzero things multiply to zero)

Actual 2D multiplication: multiply magnitudes, add angles.

Squaring a point: square its length, double its angle



The Mandelbrot Set

Process: For each point c:

Look at the ‘orbit’ of 0 when you repeat $z \to z^2 + c$

If it ‘runs away,’ color the point c based on how long it took

If it sticks around forever, color c black

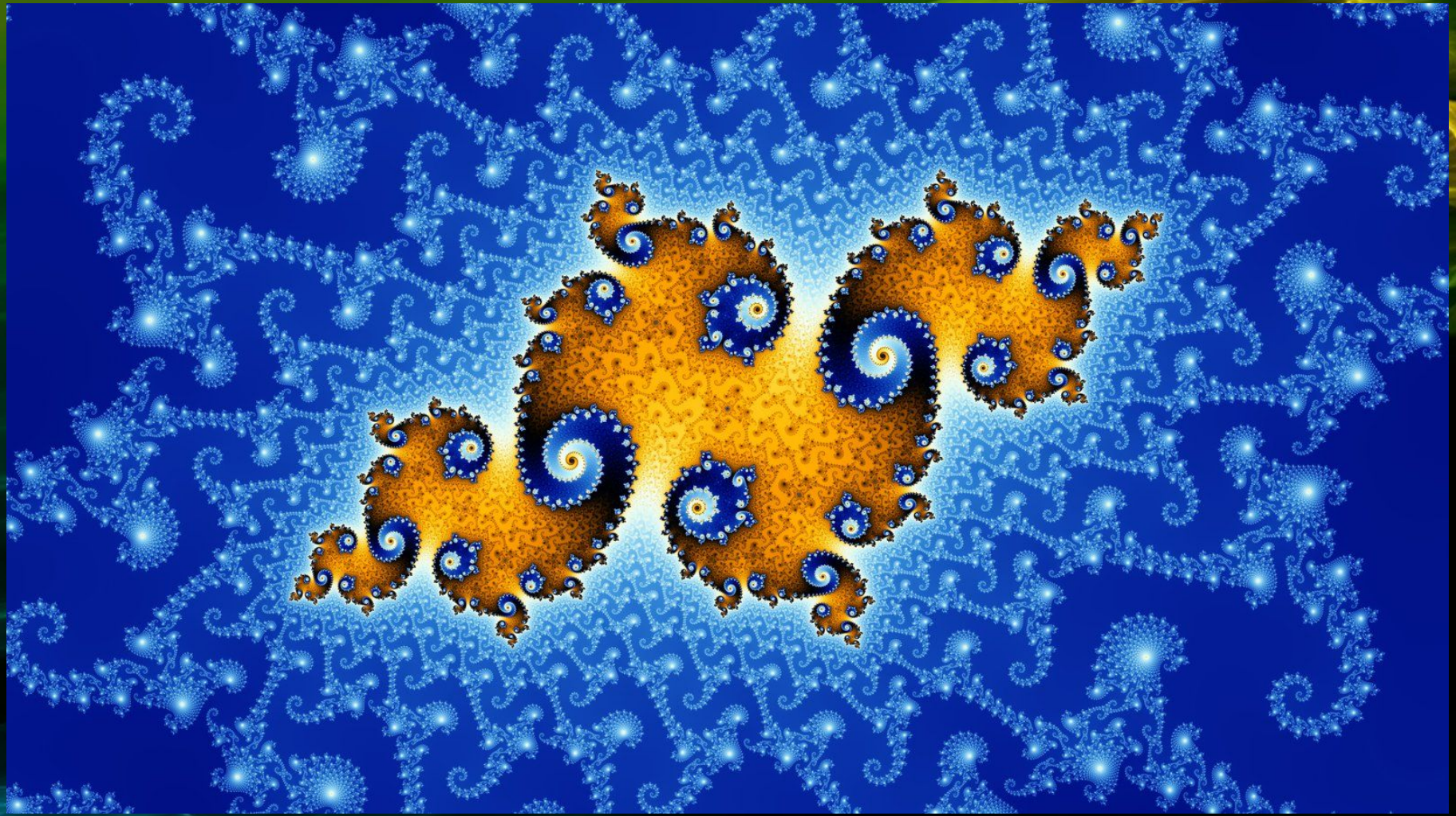
Try examples: 0, -1, 1, 1/4

Can you imagine what this will look like?

NOPE! The boundary of this shape is so curvy it’s been proven to have dimension 2!

This Shit's Amazing

http://ilde.cr

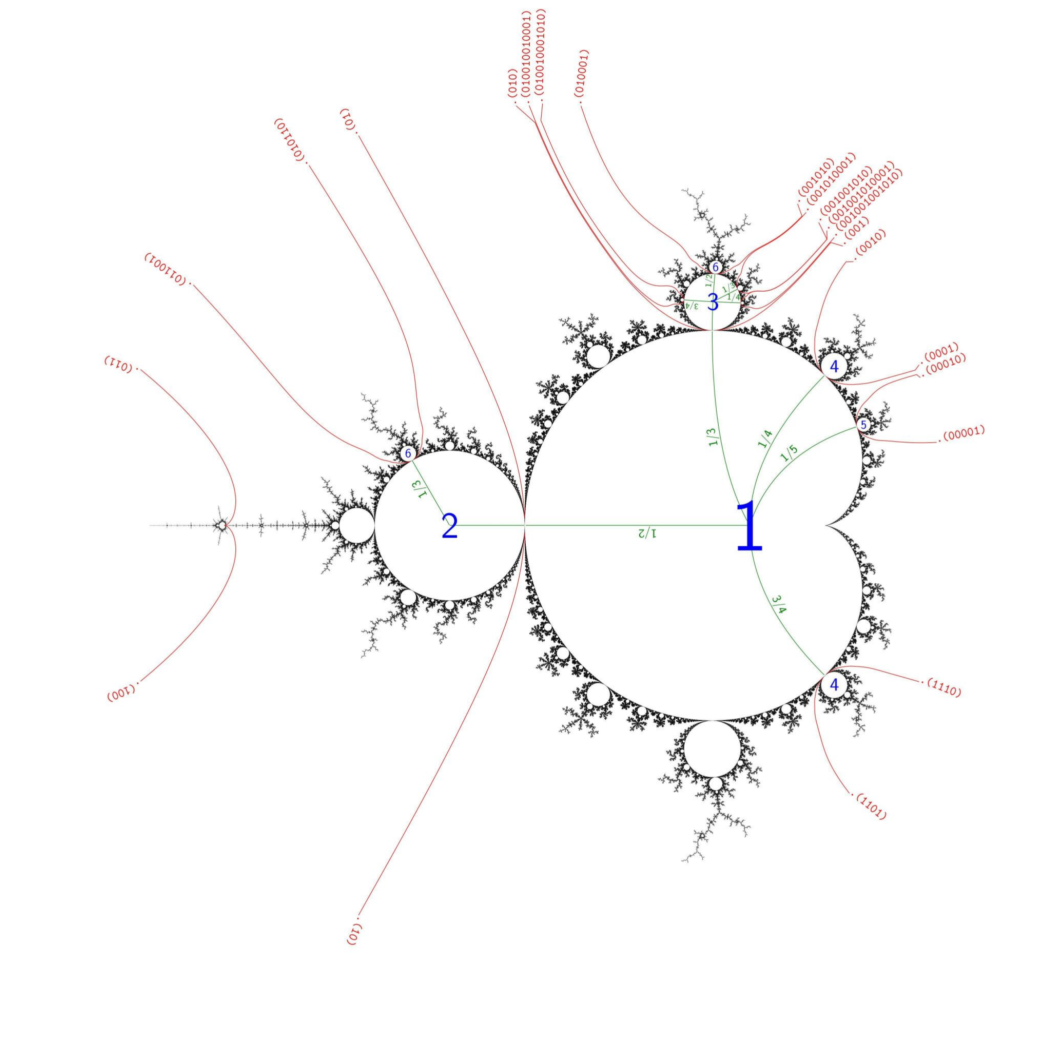


Conclusion

Fractals are a relatively new discovery, and turn out to be a good way of thinking about natural structures which look similar at many length scales/zoom levels.

Very simple rules applied many times can create amazing complexity

Mathematicians can talk about infinite iterations, leading to infinitely complex, beautiful objects in between dimensions that we can easily generate, but that no finite mind can comprehend!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3/4

Bear with me

Fractals…

- Are rooted in absolute truth
- Exist independently of our universe
- Inspire the design of the natural world
  - Share principles which are directly responsible for complex life
- Are infinitely complex, therefore incomprehensible
  - Yet born of ultimate simplicity, so partly understandable
- Are beautiful

Fractals are... divine?